μ = viscosity, g/min cm
ρ = density, g/cm^3

Subscripts and Superscripts

m = membrane
s = saturated salt solution
w = tube side water stream
1, 2 = terminal conditions
0 = wall concentration, tube side

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Experimental Studies of Natural Convection Effects On Dispersion in Packed Beds

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In two previous articles (Reejsinghani et al., 1968; Posner and Gill, 1973) it was shown that buoyancy effects in vertical open tubes can influence the extent of dispersion markedly. The purpose of this note is to study natural convection effects in packed beds which are used in a variety of industrial applications. The data cover most of the range of concentration differences (up to 26,000 ppm salt) of interest in washing the ice crystals produced in desalting by the freezing process. However, the Re_p range studied was limited to about twice the critical value ($Re_{p\ \text{critical}} \approx 3$) for fluidization to occur.

PACKED BED APPARATUS

As an extension of the experimental work done with open tubes the experimental apparatus shown in Figure 1a and 1b was set up to investigate miscible displacement in packed beds. The apparatus is designed so that a close approximation to a step change in concentration and density can be obtained at the inlet of the bed after it has been filled with the solution that is to be displaced. The area mean concentration at the outlet of the bed is monitored continuously. Both upflow and downflow experiments have been carried out to determine the effect of viscosity.

The bed is constructed of a 3.81-cm glass pipe in which 3M Company, Superbrite, Class B, narrow size distribution (70% and 30%, respectively, are retained on U.S. sieves number 40 and 60) glass beads with a density of 2.5 g/cc have been packed in the pipe and the spheres are contained by wire screens at both ends; the bed length L is 15.24 cm and the average particle diameter d_p is 0.047 cm. Thus the ratio of bed length to particle diameter is sufficient for the dispersion model to apply. The entrance section to the bed is a length of glass pipe which has been filled with a nylon mesh that flattens the velocity profile of the fluid entering the bed. Above the bed is a glass pipe spacer which is opaque except for a small region at the bottom which serves as the window for the optical system which measures the output concentration. The region above the bed also is filled with nylon mesh except for a short distance of about 2 mm in which the optical measurements are made.

The optical system consists of a light source, a G.E. No. 93 bulb powered through a step-down transformer, and a light intensity measuring system which is a No. 631A photomultiplier tube in a housing that includes two parallel slits separated by a shutter; behind the second slit is a space for colored filters and behind this is the photomultiplier tube. The photomultiplier tube is connected to an Aminco microphotometer which translates the output from the tube into light transmission data. The output from the microphotometer is monitored continuously on a 0-50 millivolt recorder.

The experimental fluids used are K_2SO_4, which forms a colorless solution, and $K_2Cr_2O_7$ which forms a colored solution. Since the dichromate solution shows maximum light absorption in the region of 4600 A, filters are placed in the photomultiplier tube housing so that only light in this region of the spectrum is measured.

ANALYSIS OF THE PACKED BED DATA

We have employed the method of Hopkins, Sheppard, and Eisenklam (1969) to determine the dispersion coefficient from the experimental data. Once one knows the concentration vs. time data at the outlet $c_2(t)$ and at the inlet $c_1(t)$ of the system then one forms the LaPlace transform of the residence time distribution which is

$$F(s) = \frac{\int_0^\infty c_{m2}(t)\, e^{-st}\, dt}{\int_0^\infty c_{m1}(t)\, e^{-st}\, dt} \tag{1}$$

For axial dispersed plug flow, the dispersion equation is given by

$$\frac{\partial c_m}{\partial t} + \frac{u_s}{\epsilon} \frac{\partial c_m}{\partial x} = k_2 \frac{\partial^2 c_m}{\partial x^2} \tag{2}$$

It can be shown, by solving for the LaPlace transform of Equation (2), using this to determine $F(s)$, and then rearranging the result, that

$$\frac{1}{\ln(1/F(s))} = \frac{\tau_L s}{\ln^2[1/F(s)]} - \frac{1}{N_{Pe}} \tag{3}$$

Therefore a plot of $-1/\ln F(s)$ vs. $s/\ln^2[F(s)]$ should be a straight line of slope τ_L and intercept $\frac{(-1)}{N_{Pe}}$. The values of $F(s)$ are found numerically from Equation (1) for various values of s by using the experimental concentration versus time curves. With N_{Pe} known k_2 can be calculated directly.

EXPERIMENTAL RESULTS ON PACKED BEDS

To ensure the accuracy of this experiment, data were collected for miscible displacements with fluids of the same density. A least squares fit shows that the correlation of the data collected in the present study doesn't differ significantly from that of Levenspiel and Bischoff (1963). Therefore we can conclude that the experimental apparatus is functioning correctly.

Experimental results are shown in Figures 2 and 3. The results of various least squares fits of the data in the form $k_2 = a\, Re_p{}^b$ are shown in Table 1 along with root mean square deviations in k_2. The data in Figure 2 are for the case of a heavier fluid below a lighter one in both upflow and downflow. It is seen that the best fit of all of the present results differs from the correlation of Levenspiel and Bischoff (1963) only at values of Re_p between 2 and 6 but the differences are not significant considering the scatter of the data is greatest in this region. The data in Table 1 indicate a tendency of the $\Delta\rho = 26{,}000$ ppm data to show slightly larger k_2 values than the other sets but this effect is small enough not to be of practical interest. However, the exponent of Re_p of 1.35 is larger than that of Levenspiel and Bischoff (1963).

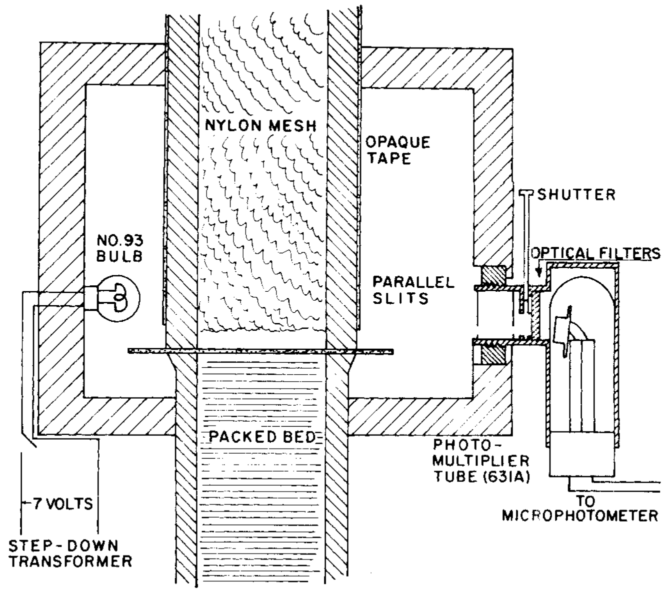

Fig. 1b. Optical system of the experimental apparatus.

TABLE 1. LEAST SQUARES FIT OF DATA IN FORM $k_2 = a\, Re_p{}^b$

	$\Delta\rho$(ppm)	a	b	Root mean square deviation in k_2
Unstable upflow	6,600	0.047	0.78	0.068
	14,400	0.041	0.92	0.085
	26,000	0.059	0.84	0.052
Unstable downflow	6,600	0.050	0.60	0.009
All unstable data		0.049	0.78	0.072
Stable upflow	6,600	0.012	1.18	0.016
	14,400	0.011	1.43	0.028
	26,000	0.017	1.47	0.013
Stable downflow	6,600	0.010	1.54	0.027
All stable data		0.013	1.35	0.027

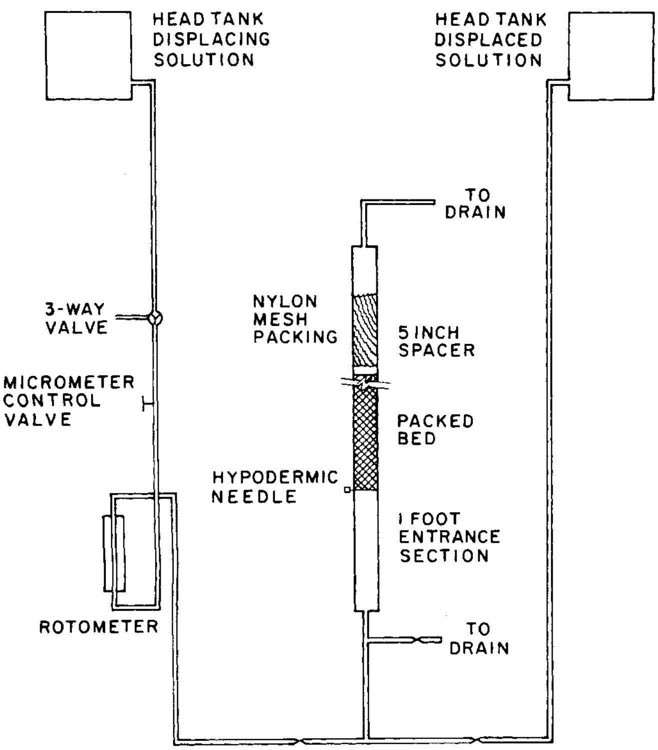

Fig. 1a. Flow schematic of the experimental apparatus.

Apparently any alteration in the velocity profile in the microstructure, which may occur as it does in open tubes, is attenuated strongly by the complex nature of the flow path in a packed bed and the effect of density differences in the stable configuration is unimportant.

When the displacement involves the lighter fluid below the heavier one, in either upflow or downflow, the situation is hydrodynamically unstable with respect to buoyancy forces. Figure 3 illustrates this case, and it is seen in Table 1 that the values of a are almost four times larger in the unstable cases than the stable ones. On the other hand, a is only 25% larger for the largest density difference in Figure 3 than for the smallest one. These data for the unstable case scatter more than those in Figure 2; the root mean square deviation of k_2 is about 2.5 times larger for the data in Figure 3. This scatter is due to the interaction caused by bringing fluids of different density together at $t = 0$ in an unstable configuration and to the empty tube natural convection effect above the bed which is significant even though the region between the bed and the nylon mesh above it is only 2 mm long. The unstable density gradient in the open tube between the bed and the nylon mesh probably is the most important cause of scatter. Another cause of scatter is the small rearrangement of the bed that may occur from run to run, particularly at

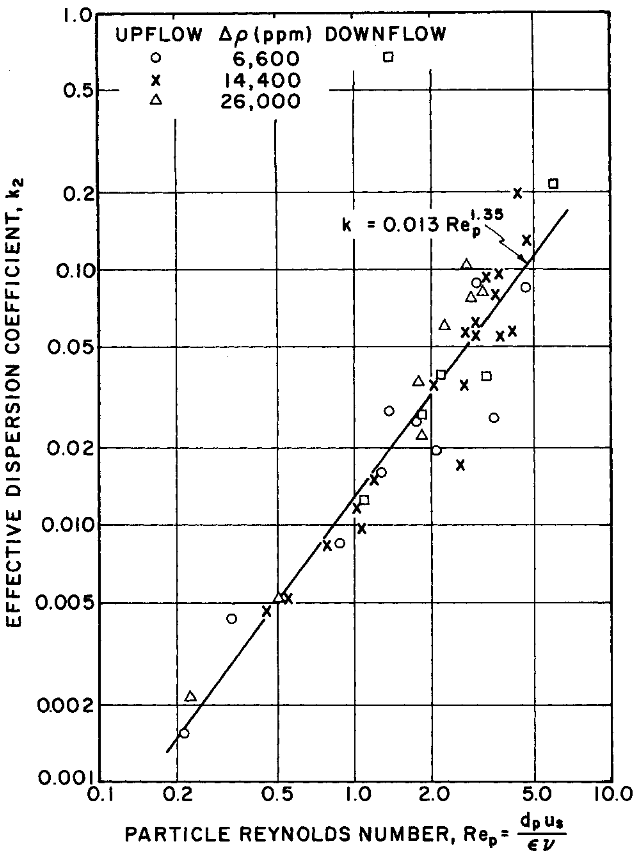

Fig. 2. Stable case with heavier fluid on bottom, k_2 vs. Re_p, with $\epsilon = 0.38$

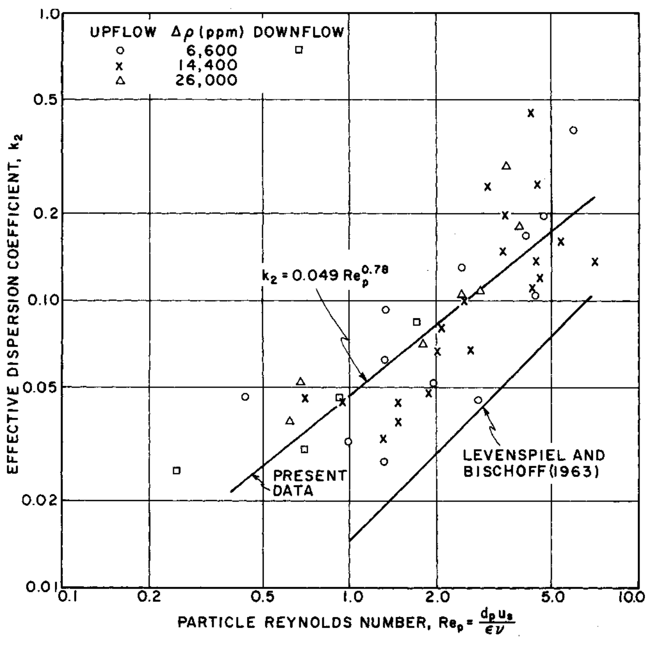

Fig. 3. Unstable case with lighter fluid on bottom, k_2 vs. Re_p, with $\epsilon = 0.38$.

higher values of Re_p. (Note that the data in Figure 2 scatter most above Re_p critical.) Also, note that the root mean square for the downflow data in Figure 3 is very small.

Despite their scatter, the data in Figure 3 show clearly that natural convection increases the dispersion coefficient significantly. For example, at Re_p equal 1, the dispersion coefficient is 3.5 times that without density differences; at $Re_p = 7$ this ratio is about 2. Thus, one sees that for the same density difference, or Grashof number, the effect of natural convection on the dispersion coefficient is larger at smaller values of the particle Reynolds number.

In upflow with the less viscous fluid displacing the more viscous one, the viscosity effects are unfavorable and may cause some fingering which would increase dispersion. To establish that viscosity effects were negligible, down flow experiments were run. For the downflow results in Figures 2 and 3, the lighter fluid is still on the bottom but now the more viscous fluid is displacing the less viscous one; this corresponds to a favorable viscosity ratio which inhibits fingering and therefore one would not expect viscosity effects to influence the extent of dispersion in these downflow experiments. It is seen that there are no significant differences between the upflow and downflow experiments. Thus we can conclude that viscosity effects were not important in these experiments and the enhanced dispersion observed in upflow with the lighter fluid displacing the heavier one is due to buoyancy forces induced by density differences.

In conclusion, the data show that the enhancement of the dispersion coefficient by buoyancy effects is much greater in open tubes than the present results show for packed beds. Considerable elongation of the velocity profile in open tubes causes larger differences in residence times across the flow and this increases dispersion very significantly. The bed, because of its complex nature, prevents velocity distributions from developing in the way they do in open tubes and therefore it tends to inhibit dispersion.

ACKNOWLEDGMENT

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NOTATION

c_m = average concentration
d_p = particle diameter
k_2 = dispersion coefficient
L = bed length
$N_{Pe} = \dfrac{u_s L}{k_2 \epsilon}$
$Re_p = \dfrac{d_p u_s}{\epsilon \nu}$
s = transform parameter
t = time
u_s = superficial velocity
x = axial distance
ϵ = void fraction
ν = kinematic viscosity
ρ = solute concentration, ppm
$\tau_L = L\epsilon/u_s$

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